

QgeoidMEDE2026 Calculation

Technical report

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1 Available Gravity Data

1.1 Spatial Distribution of the Data

The area covered by the model is located between $281^{\circ}34'52'' \leq \text{longitude} \leq 287^{\circ}15'52''$ and $3^{\circ}21'34''\text{N} \leq \text{latitude} \leq 9^{\circ}2'34''\text{N}$. This extension area was defined to support the computation of an IHRF geopotential number at the permanent CORS geodetic monitoring station. Figure 1 shows the gravity data available for the computation of the quasigeoid model within this extension area.

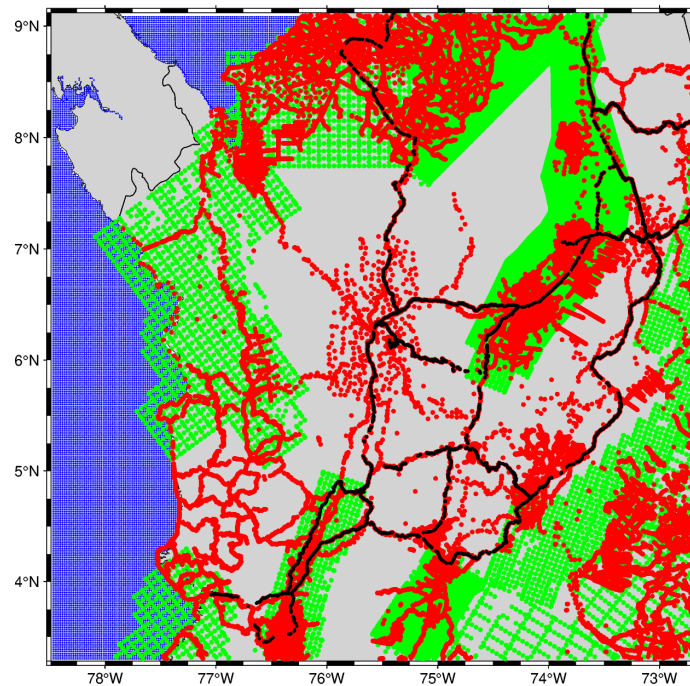


Figure 1: Gravity data: blue (DTU21Gra data), red (terrestrial observations), green (airborne observations), and black (Leveling/GNSS observations used for validation).

For the computation of the quasigeoid model for the MEDE station, a total of **20704** airborne gravity observations, **30482** terrestrial gravity observations, and **18995** satellite altimetry-derived gravity observations were used.

The validation dataset consists of geometric height anomalies derived from geodetic leveling and GNSS positioning. The normal heights computed from geodetic leveling and gravity data were adjusted using geopotential numbers. A total of 2790 validation points are available. In particular, the Loboguerrero-Pereira leveling line comprises 150 benchmarks, which were used for the validation of the QgeoidMEDE2026 quasigeoid model.

2 Remove Procedure

2.1 Terrestrial Gravity Data

This section presents the results obtained from the remove procedure applied to the terrestrial gravity data using the **XGM2019** model (0–719 degree and order (d/o)), **dv_ell_Earth_2014** (720–2159 d/o), and **ERTM2160**. The results are shown in Figure 2.

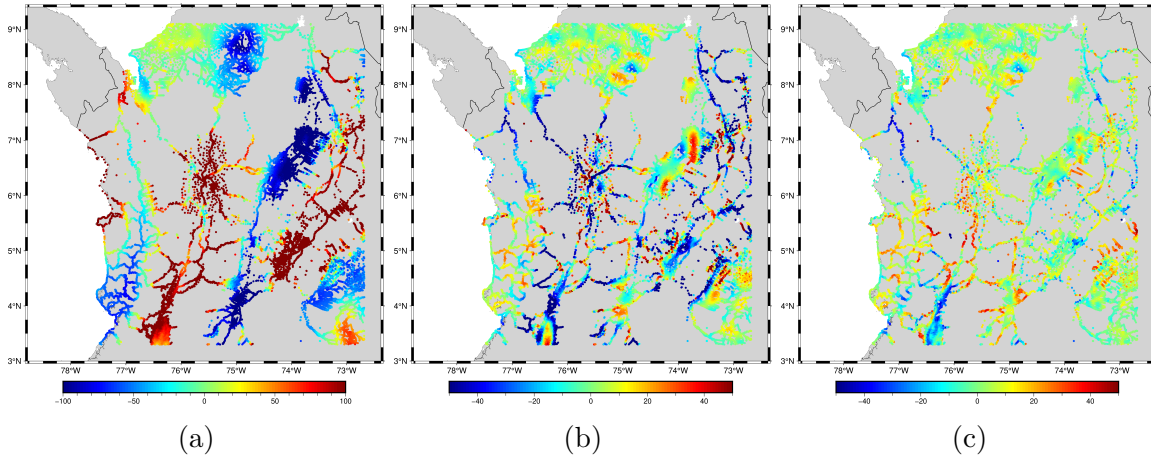


Figure 2: (a) Gravity disturbances observations (δg), (b) gravity disturbances after removing the long-wavelength component ($\delta g - \delta g_{GGM}$), and (c) residual gravity disturbances ($\delta g - \delta g_{GGM} - \delta g_{Gtopo}$).

2.2 Airborne Gravity Data

This section presents the results obtained from the remove procedure applied to the airborne gravity data using the **XGM2019** model (d/o 0-719) and **dv_ell_Earth2014** (d/o 720-5480). The results are shown in Figure 3.

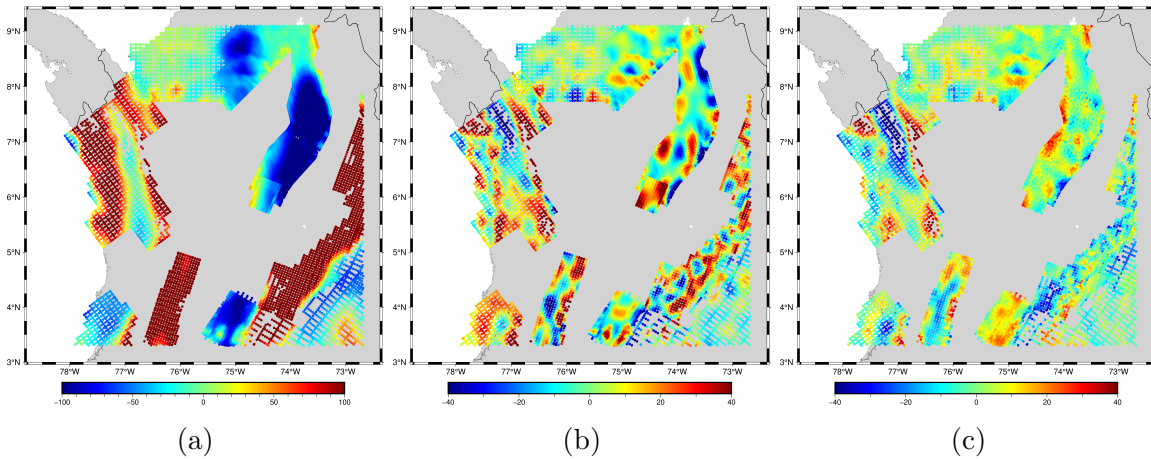


Figure 3: (a) Gravity disturbances observations (δg), (b) gravity disturbances after removing the long-wavelength component ($\delta g - \delta g_{GGM}$), and (c) residual gravity disturbances ($\delta g - \delta g_{GGM} - \delta g_{Gtopo}$).

2.3 DTU21Gra Data

This section presents the results obtained from the remove procedure applied to the DTU21Gra gravity data using the **XGM2019** model (d/o 0-719) and **dv_ell_Earth2014** (d/o 720-5480). The results are shown in Figure 4.

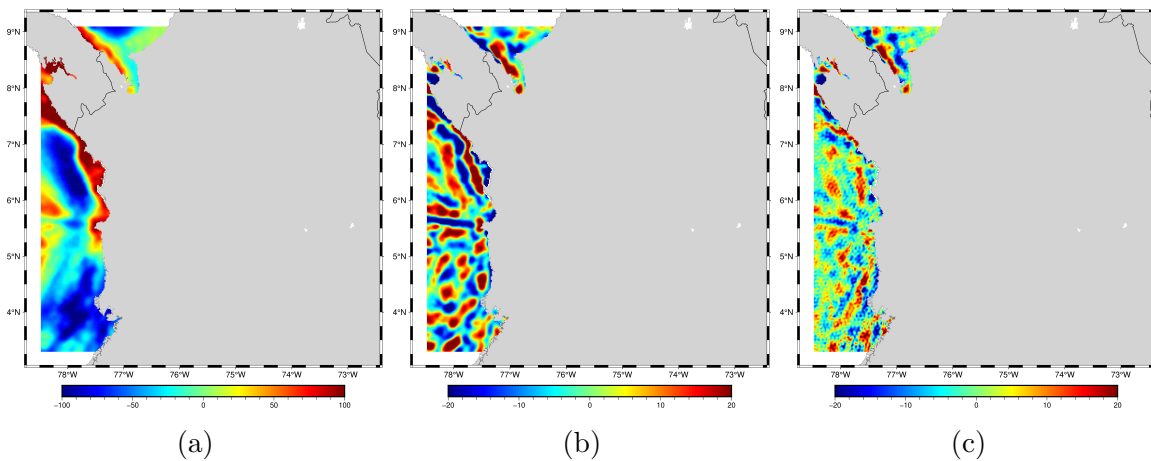


Figure 4: (a) Gravity anomalies observations (Δg), (b) gravity anomalies after removing the long-wavelength component ($\Delta g - \Delta g_{GGM}$), and (c) residual gravity anomalies ($\Delta g - \Delta g_{GGM} - \Delta g_{Gtopo}$).

2.4 Descriptive Statistics of the Remove Procedure

This section summarizes the basic descriptive statistics of the remove procedure. Table 1 presents the statistics obtained by the geoid working group.

		Minimum (mGal)	Maximum (mGal)	Mean (mGal)	σ (mGal)
Terrestrial	δg	-140.9281	349.5970	7.6078	86.3383
	$\delta g - \delta g_{\text{GGM}}$	-225.0013	145.6824	-6.1703	28.7376
	$\delta g - \delta g_{\text{GGM}} - \delta g_{\text{topo}}$	-183.4536	69.3697	2.2409	12.4478
Airborne	δg	-137.6088	350.5275	-15.1894	82.5647
	$\delta g - \delta g_{\text{GGM}}$	-85.4464	95.9191	1.8029	17.0106
	$\delta g - \delta g_{\text{GGM}} - \delta g_{\text{topo}}$	-53.4928	71.9458	1.4182	10.9630
Altimetry	Δg	-117.9630	178.3950	-17.6845	56.1105
	$\Delta g - \Delta g_{\text{GGM}}$	-54.6952	62.1654	0.0775	13.0939
	$\Delta g - \Delta g_{\text{GGM}} - \Delta g_{\text{topo}}$	-46.3637	52.3618	0.1599	8.7735

Table 1: Descriptive statistics of the gravity data before and after removing the long-wavelength component (GGM) and the topographic effect (Gtopo).

3 Parameter Estimation and Regularization of the System of Equations

3.1 Variance Component Estimation (VCE)

This section presents the convergence of the variance component estimation for each gravity data set ($\sigma_1^2, \dots, \sigma_L^2$, where L denotes the total number of gravity data types). The convergence of the variance factor associated with the regularization parameter, σ_μ^2 , is also shown.

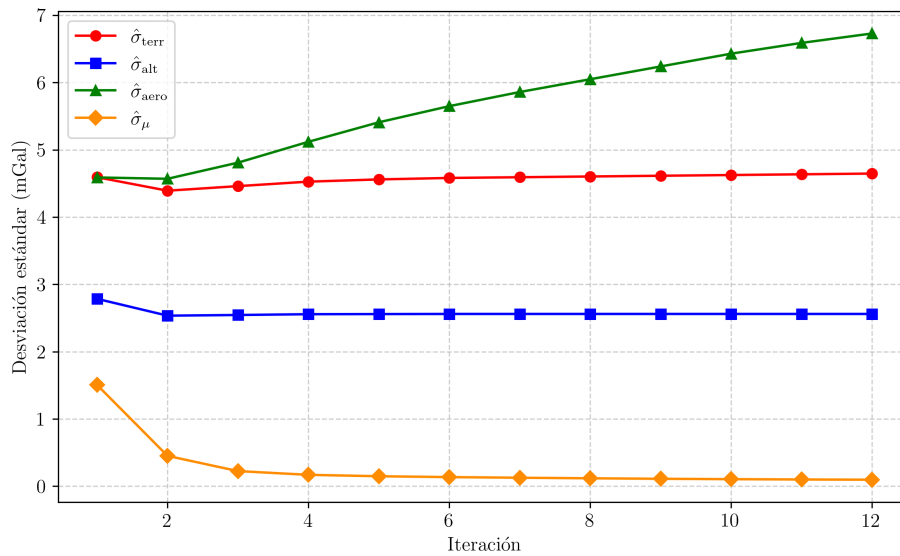


Figure 5: Convergence of the estimated variance components ($\hat{\sigma}_i$).

The iterative variance component estimation converged to standard deviations of **4.64 mGal** for the terrestrial gravity data, **2.56 mGal** for the satellite altimetry data, and **6.73 mGal** for the airborne gravity data.

3.2 L-Curve

The optimal Tikhonov regularization parameter was selected following the methodology proposed by [Hansen and O'Leary \(1993\)](#). Figure 6 presents the selected regularization parameter λ together with the L-curve, which depicts the relationship between the residual 2-norm and the regularization 2-norm.

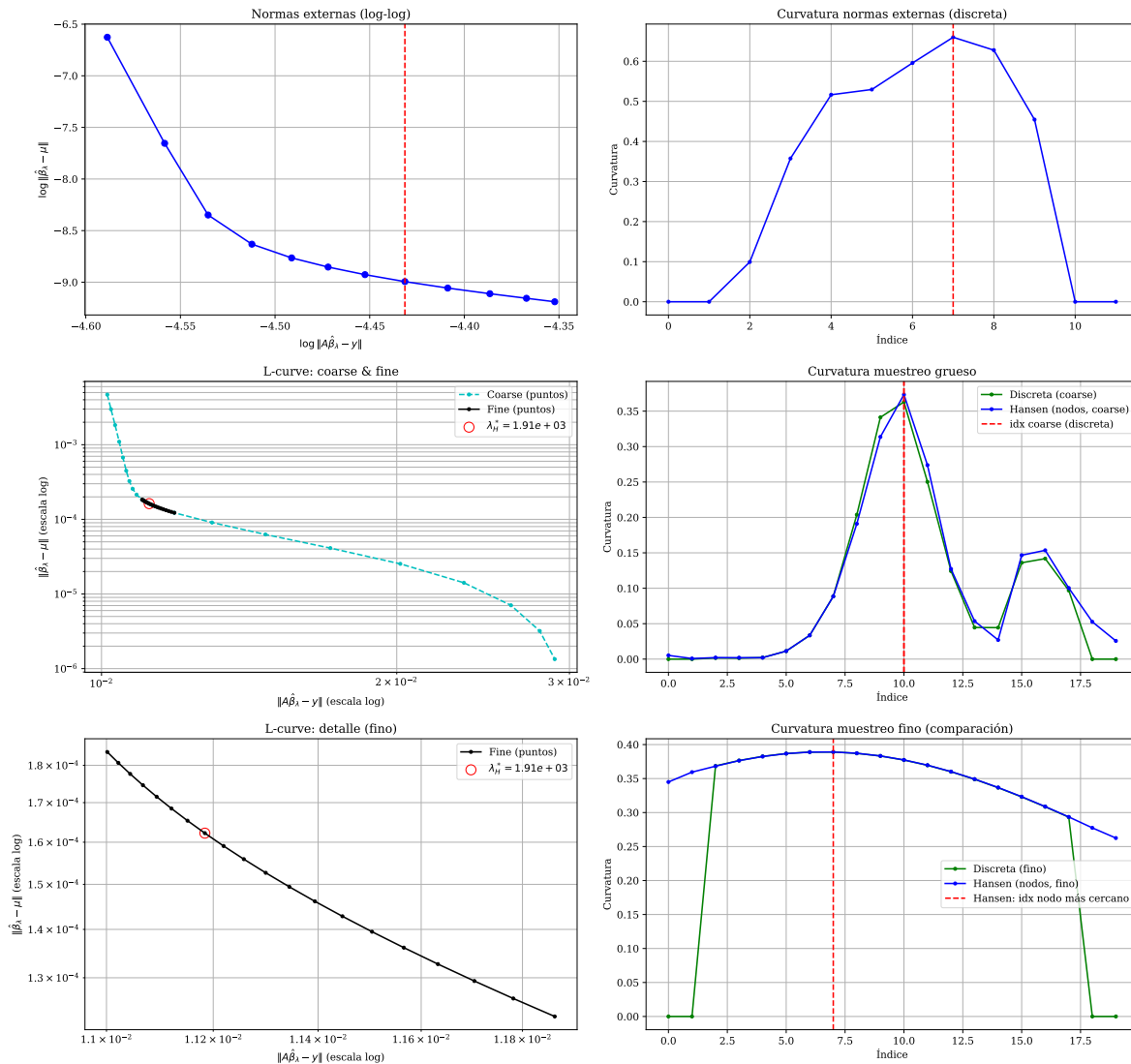


Figure 6: L-curve used for selecting the Tikhonov regularization parameter.

According to the L-curve criterion, the optimal Tikhonov regularization parameter is $\lambda = 1906$.

3.3 Estimated Parameters ($\hat{d}$, $\hat{\sigma}_d$)

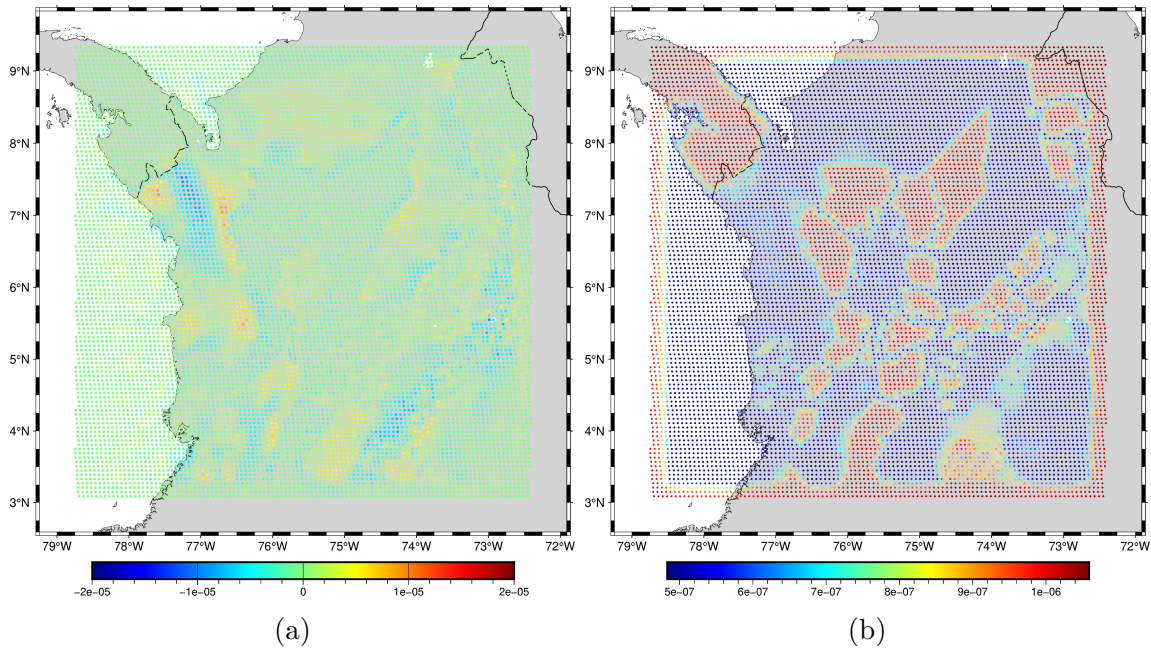


Figure 7: (a) Estimated parameters ($\hat{d}$), (b) standard deviation of the estimated parameters ($\hat{\sigma}_d$).

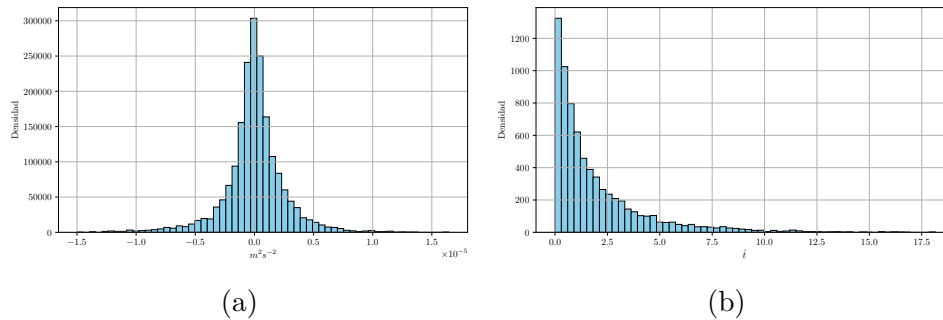


Figure 8: (a) Histogram of the estimated parameters ($\hat{d}$), (b) distribution of the t -statistics ($\hat{t}$).

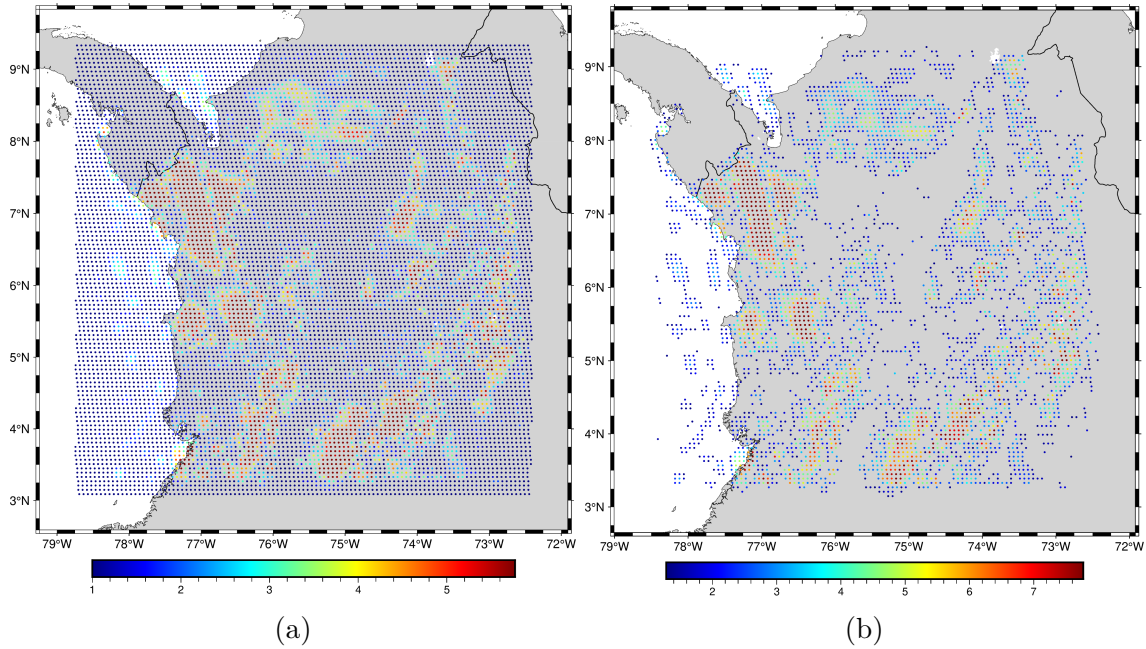


Figure 9: (a) t -statistics ($\hat{t}$), (b) statistically significant parameters ($\hat{t} \geq t_{\text{crit}}$).

A total of 11027 parameters were estimated on the Reuter grid. Applying a hypothesis test based on the t -statistic at a significance level of 10%, 3534 parameters were found to be significantly different from zero, corresponding to approximately 32.05% of the total estimated parameters. The hypothesis test is formulated as

$$H_0 : d = 0 \quad \text{vs.} \quad H_a : d \neq 0. \quad (1)$$

The test statistic is compared with the critical t -value according to

$$\hat{t} = \frac{|\hat{d}|}{\hat{\sigma}_d} \quad \text{vs.} \quad t_{(\text{dof}, p)}. \quad (2)$$

3.4 Estimation of the Disturbing Potential ($\hat{T}, \hat{\sigma}_T$)

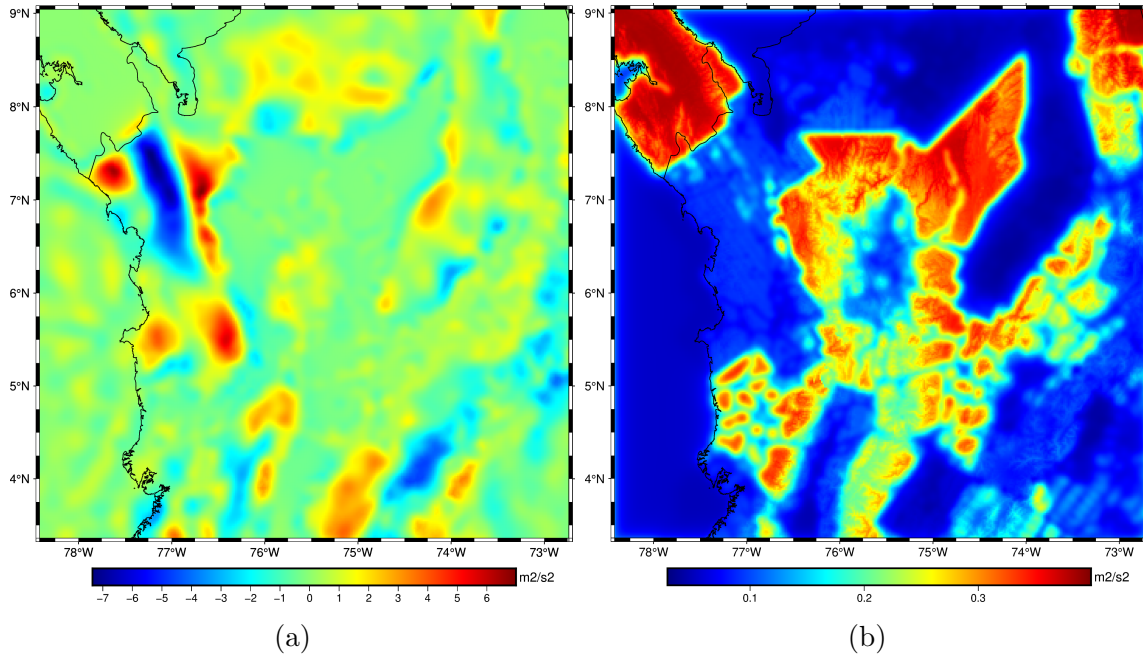


Figure 10: (a) Estimated residual disturbing potential ($\hat{T}$), (b) standard deviation of the disturbing potential ($\hat{\sigma}_T$).

For the estimation of the disturbing potential, the design matrix A was constructed using the following kernel:

$$B(x, x_k) = \sum_{n=0}^{N_{max}} \frac{2n+1}{4\pi} \left(\frac{R}{r} \right)^{n+1} B_n P_n(\cos \psi). \quad (3)$$

Once the parameter vector $\hat{d}$ has been estimated, the disturbing potential is computed as

$$\hat{T} = A\hat{d}, \quad (4)$$

The covariance propagation for the disturbing potential is given by

$$D(\hat{T}) = AD(\hat{d})A^T. \quad (5)$$

3.5 Estimation of the Height Anomaly ($\hat{\zeta}, \hat{\sigma}_{\zeta}$)

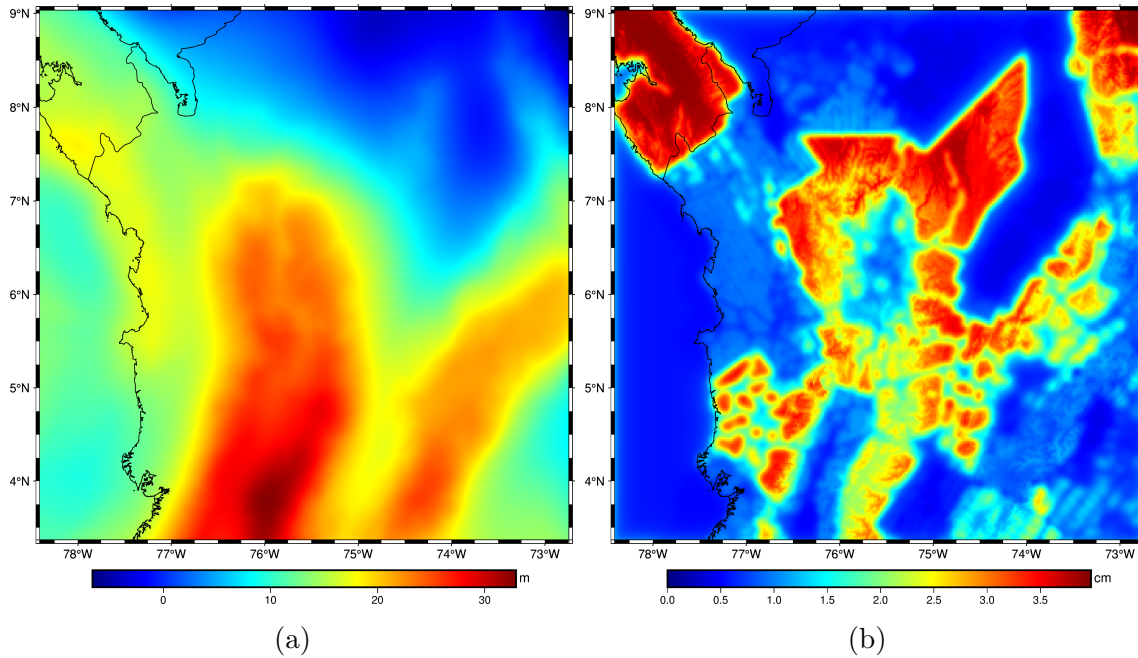


Figure 11: (a) QgeoidMEDE2026 ($\hat{\zeta}$), (b) standard deviation of the quasigeoid model ($\hat{\sigma}_{\zeta}$).

Once the disturbing potential $\hat{T}$ has been estimated, the quasigeoid height anomaly is computed using Bruns' theorem, as described in [Hofmann-Wellenhof and Moritz \(2006\)](#). Subsequently, the previously removed gravity field signal is restored to the gravimetric quasigeoid model using the following models: **XGM2019** (d/o 0-719), **dv_ell_Earth2014** (d/o 720-2159), and **ERTM2160** (d/o 2160 to approximately 80,000).

To ensure compliance with the International Height Reference System (IHRs) standards, the zero-degree term of the height anomaly for the QgeoidMEDE2026 model was computed using

$$\zeta_0 = \frac{KM_{GGM} - KM_{GRS80}}{r_p \cdot \gamma_Q} - \frac{W_0 - U_0}{\gamma_Q}. \quad (6)$$

4 GNSS/Leveling Validation

4.1 Absolute Validation

The absolute validation was performed by comparing, point by point, the gravimetric height anomalies of each quasigeoid model (ζ) with the geometric height anomalies ($\zeta^{\text{Leveling/GNSS}}$) derived from geodetic leveling adjusted by geopotential numbers (leveling and gravity) together with GNSS positioning. A total of 2794 benchmarks were used in the validation.

-	Minimum (cm)	Maximum (cm)	Mean (cm)	σ (cm)
QgeoidMEDE2026	-34.25	53.58	0.8	16.38
QgeoidCOL2023	-36.64	56.82	0.9	17.61
SAM2023	-60.37	48.53	-0.7	20.72
XGM2019	-63.93	72.26	0.4	26.17

Table 2: Descriptive statistics of the absolute validation.

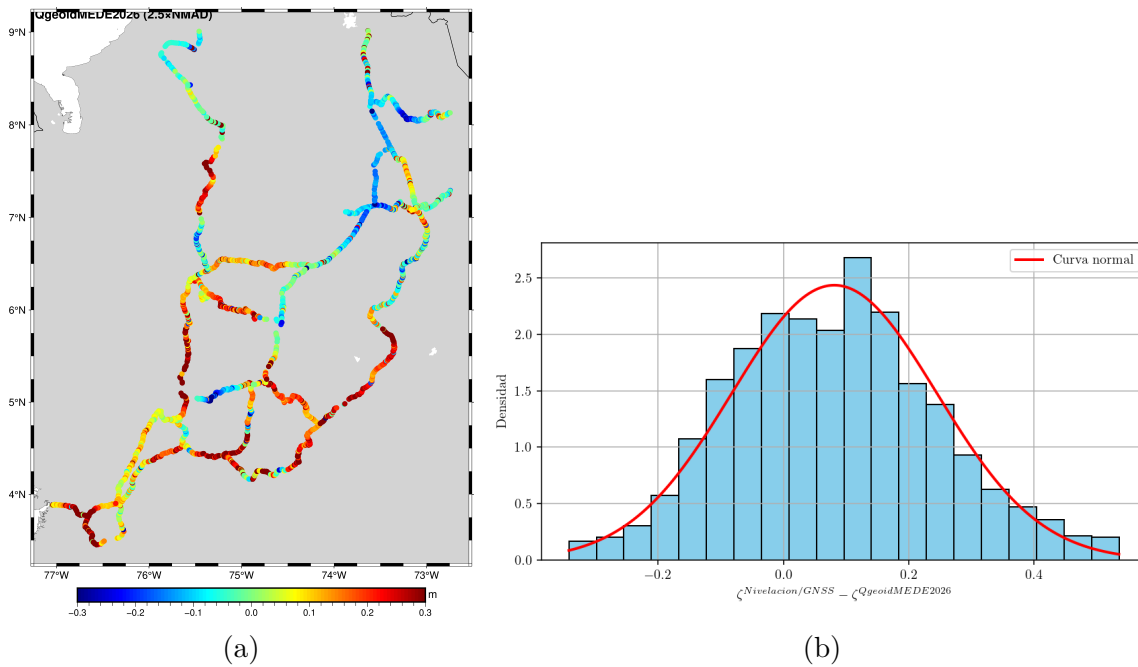


Figure 12: Differences between $\zeta^{\text{Leveling/GNSS}}$ and $\zeta^{\text{QgeoidMEDE2026}}$: (a) spatial distribution and (b) histogram.

Additionally, the most recent leveling line, surveyed in 2024 and complemented with gravity observations at each benchmark, extends from Loboguerrero to Pereira. The absolute

validation of the quasigeoid models along this leveling line yielded the following results.

-	Minimum (cm)	Maximum (cm)	Mean (cm)	σ (cm)
QgeoidMEDE2026	0.68	20.61	8.90	4.11
QgeoidCOL2023	-9.95	39.40	15.78	11.92
SAM2023	-32.53	20.99	-6.49	11.74
XGM2019	-31.06	69.59	-8.49	31.06

Table 3: Descriptive statistics of the absolute validation along the Loboguerrero–Pereira leveling line.

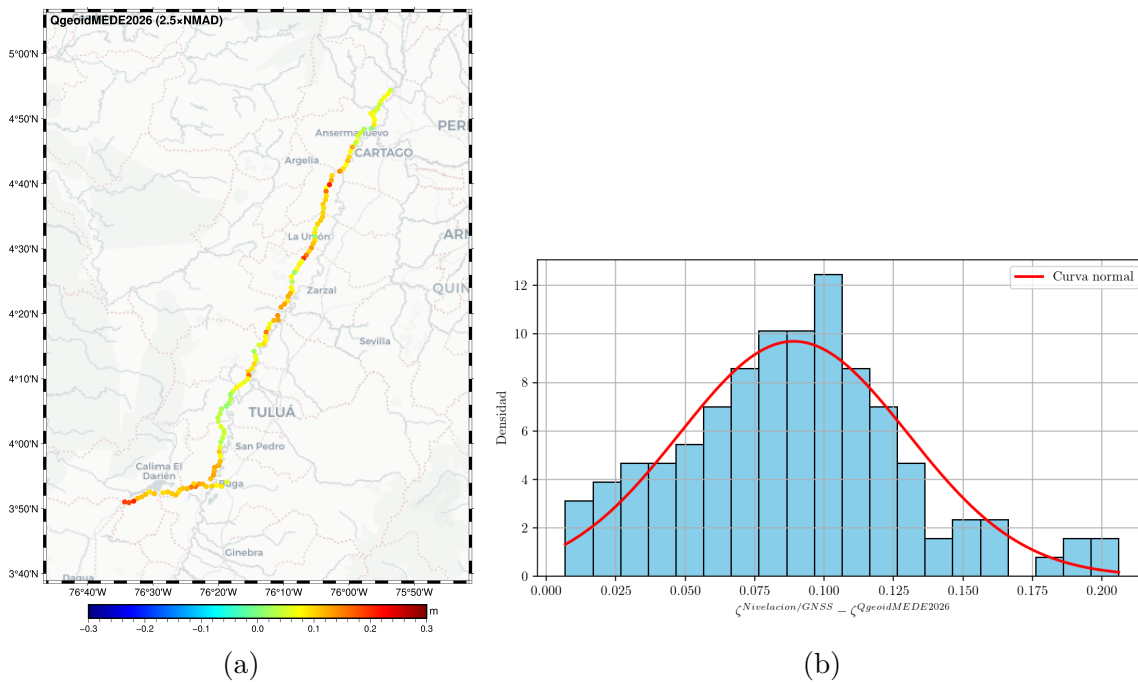


Figure 13: Differences between $\zeta^{\text{Leveling/GNSS}}$ and $\zeta^{\text{QgeoidMEDE2026}}$ along the Loboguerrero–Pereira leveling line: (a) spatial distribution and (b) histogram.

4.2 Relative Validation

The relative validation was carried out using Equation 7 following Liu et al. (2024). This validation evaluates the capability of the quasigeoid models to reproduce the relative height anomaly differences (slopes) observed from the GNSS/leveling data for baseline lengths of up to 700 km.

$$\Delta\zeta_{ij} = \left(\zeta_i^{\text{GNSS/Leveling}} - \zeta_j^{\text{GNSS/Leveling}} \right) - \left(\zeta_i^{\text{QgeoidMEDE2026}} - \zeta_j^{\text{QgeoidMEDE2026}} \right). \quad (7)$$

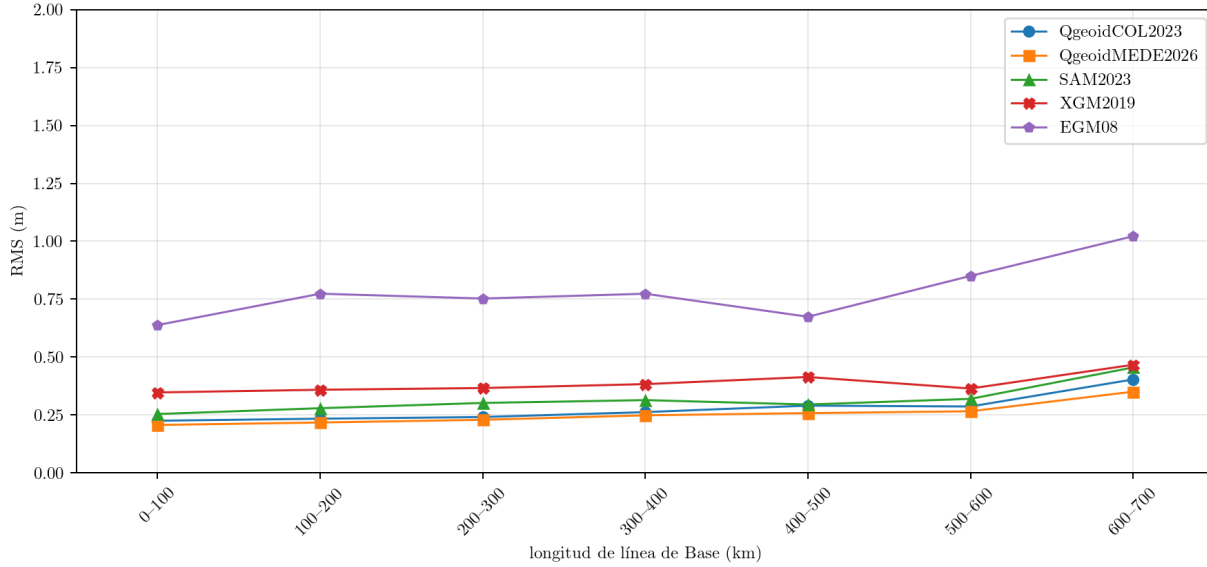


Figure 14: Relative validation of regional quasigeoid models.

The results indicate nearly constant performance in reproducing the relative differences between geometric height anomalies (GNSS/Leveling) and gravimetric height anomalies (quasigeoid models) for baseline lengths of up to 700 km. The regional and national quasigeoid models, QgeoidMEDE2026 and QgeoidCOL2023, exhibit the lowest RMS values, indicating better consistency with the GNSS/leveling observations than the remaining models. QgeoidMEDE2026 achieves an average RMS of **23.14 cm**, while QgeoidCOL2023 yields an average RMS of **24.84 cm**. Both models clearly demonstrate the benefits of regional refinement of the global geopotential model XGM2019.

As in the absolute validation, the relative performance of the models was also evaluated along the Loboguerrero–Pereira leveling line. RMS values were computed for benchmark pairs separated by baseline lengths of up to 120 km. QgeoidMEDE2026 achieved a weighted average RMS of approximately **5.97 cm**, whereas QgeoidCOL2023, SAM2023, and XGM2019 yielded weighted average RMS values of **16.30 cm**, **16.09 cm**, and **42.47 cm**, respectively. These results clearly highlight the improvement provided by the regional refinement applied to the XGM2019-based quasigeoid models.

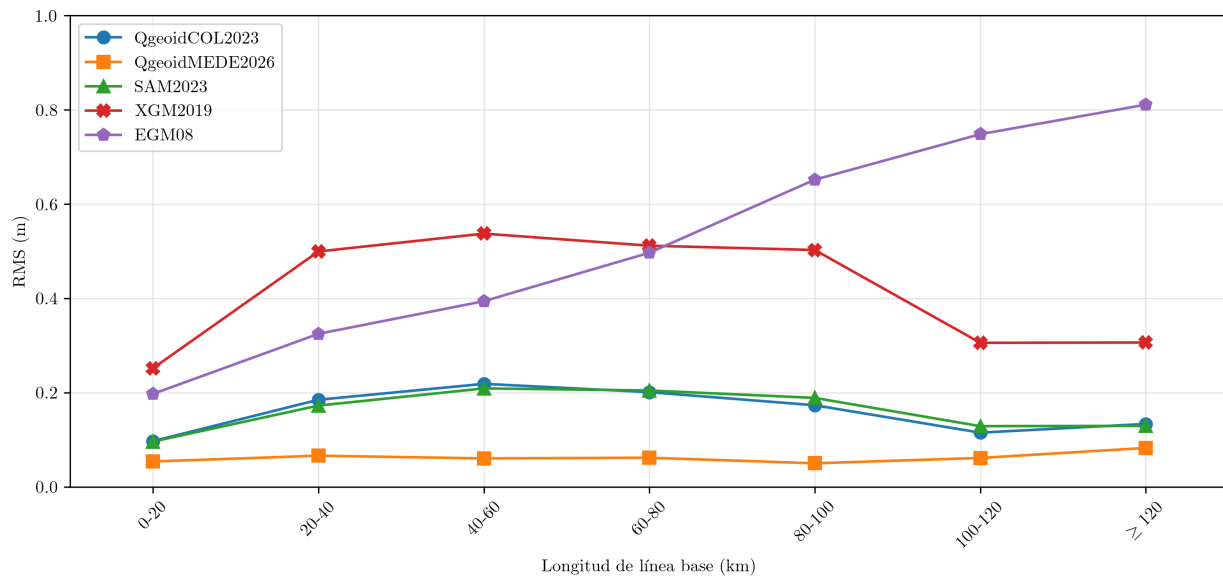


Figure 15: Relative validation of regional quasigeoid models along the Loboguerrero–Pereira leveling line.

5 Comparison with Other Models

A spherical harmonic synthesis of the XGM2019, GOSG02S, SGG-UGM-2, and EGM2008 models was performed at the grid nodes of the QgeoidMEDE2026 model. In addition, the grid-based models SAM2023 and QgeoidCOL2023 were interpolated onto the same grid. The resulting differences are shown in Figure 16.

The smallest differences with respect to QgeoidMEDE2026 are observed for the XGM2019 model combined with the spherical harmonic expansion of the $dv_ell_Earth2014$ model and the ERTM2160 topographic model, which constitute the background models used in the computation of QgeoidMEDE2026. Similarly, QgeoidCOL2023 exhibits high agreement with QgeoidMEDE2026, reflecting the close relationship between the two regional quasigeoid models.

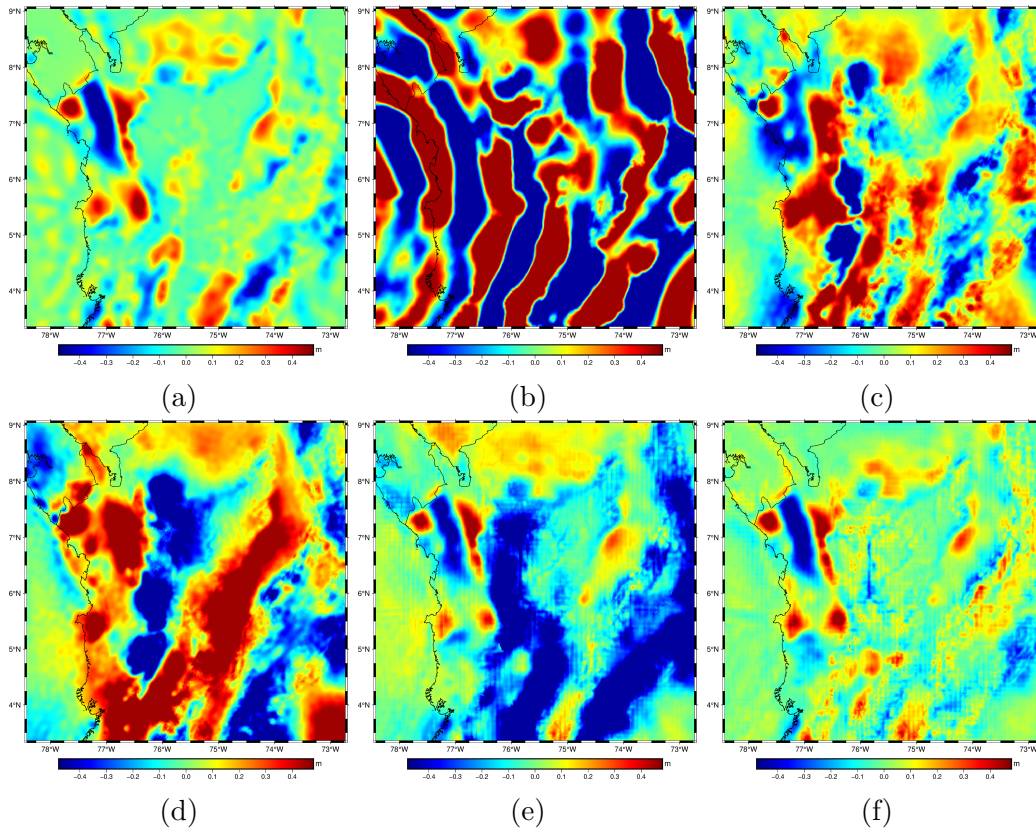


Figure 16: Differences between QgeoidMEDE2026 and (a) XGM2019, (b) GOSG02S, (c) SGG-UGM-2, (d) EGM2008, (e) SAM2023, and (f) QgeoidCOL2023.

	Minimum (cm)	Maximum (cm)	Mean (cm)	σ (cm)
XGM2019	-75.15	71.42	0.05	12.66
GOSG02S	-258.33	332.05	2.28	87.31
SGG-UGM-2	-269.61	120.58	2.88	26.73
EGM2008	-312.36	161.43	5.91	38.48
SAM2023	-153.35	73.84	-14.89	27.97
QgeoidCOL2023	-75.45	70.37	0.70	11.75

Table 4: Descriptive statistics of the differences between QgeoidMEDE2026 and other global and regional geopotential models.

6 Least-Squares Adjustment Residuals

This section presents the residuals obtained from the least-squares adjustment for each gravity data set (Figure 17).

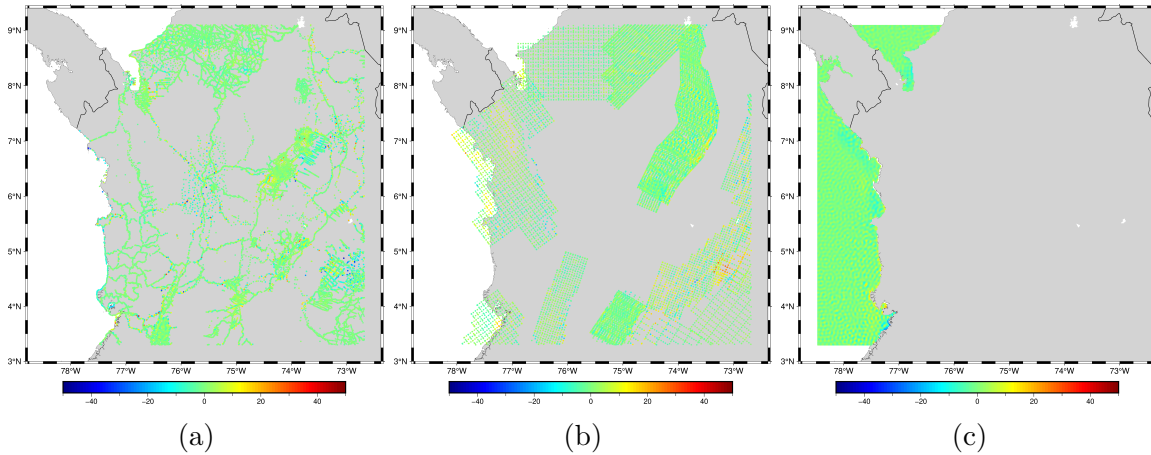


Figure 17: Residuals of the least-squares adjustment for the gravimetric observations: (a) terrestrial gravity data, (b) airborne gravity data, and (c) satellite altimetry data.

	Minimum (mGal)	Maximum (mGal)	Mean (mGal)	σ (mGal)
Terrestrial	-44.96	45.51	-0.20	4.64
Airborne	-23.95	28.61	0.21	4.22
Satellite Altimetry	-15.77	14.76	-0.02	2.86

Table 5: Descriptive statistics of the least-squares adjustment residuals.

Figure 17 illustrates the spatial distribution of the adjustment residuals, highlighting the regions where the estimated parameters successfully explain the variability of the gravity observations. In particular, the green-colored areas correspond to locations with the smallest differences between the observed gravity quantities ($\Delta\delta g$) and their estimated values ($\Delta\hat{\delta}g$), indicating the best agreement between the observations and the adjusted model.

References

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- Hofmann-Wellenhof, B. and Moritz, H. (2006). *Physical Geodesy*. Springer.
- Liu, Q., Schmidt, M., Sánchez, L., et al. (2024). High-resolution regional gravity field modeling in data challenging regions for the realization of geopotential-based height systems. *Earth Planets Space*, 76(35).