IMPROVEMENT OF THE GRAVIMETRIC GEOID IN THE UKRAINE AREA USING ABSOLUTE GRAVITY DATA

A.N. Marchenko, O.A.Abrikosov

Faculty of Geodesy, State University “Lviv Polytechnic”, S.Bandera St. 12, 290646 Lviv, Ukraine

P.O.Romanishin

Main Administration of Geodesy, Cartography and Cadastre, Popudrenka St. 54, 252660 Kiev, Ukraine

Summary. The gravimetric geoid in the Ukraine area was determined by two steps. At the first one the preliminary geoid solution was derived by means of the Sequential Multipole Analysis technique applied to the 1748 free-air gravity anomalies averaged by blocks 5’×7.5’ and 20’×30’. At the second step the preliminary solution was tied to the fundamental absolute gravity network of Ukraine taking the points of this network as the control ones. The final geoid solution was derived from total least-squares re-adjustment of the previously obtained set of the radial multipoles. The absolute gravity data were included into the re-adjustment as the additional conditions because these data have essentially higher accuracy than the gravity anomalies used at the first step. Contribution of the absolute gravity data in the geoid results is analysed.

Introduction

One of the main tasks of the West Ukraine Geoid Project (WUGP) is construction of *analytical models* of regional and local gravity field for fast and precise geoid computations. Preliminary construction of such model was performed (Marchenko and Abrikosov 1994a) on the ground of terrestrial free-air mean gravity anomalies approximation by sets of non-central radial multipoles in the frames of the *sequential multipole analysis* **(**SMA**)** technique (Marchenko and Abrikosov 1994b). Clearly, that best results may be found by the way of different - heterogeneous data processing; in particular, by both absolute gravity observations and fundamental gravity network stations data using.

With the aim of further improvement of the Ukraine geoid model the following problems are proposed for investigations:

1. Simultaneous processing of Geosat altimeter data in the Black and Azov Seas and mean gravity data in the Ukraine area.
2. Simultaneous adjustment of the preliminary geoid version and the data of the fundamental gravity network of Ukraine taking some points of this network as the control ones.

A more detailed discussion of these problems is given below.

Method

At an external point P we will represent the disturbing gravity potential T by certain sum of potentials of radial multipoles

$$T(P) = \sum_{i=1}^{K} \frac{m_i^{(n)}}{r_P^{n+1}} v_n^i(P) \quad , \tag{1}$$

where $v_n^i(P)$ is the dimensionless potential of the n-th degree radial multipole located at the point i of the Bjerhammar sphere interior; $m_i^{(n)}$ is the moment of the multipole; r_P is the geocentric distance of the point P. Note here, that each multipole has its own degree $n=n_i$. Representation (1) leads to the following relationships for geoidal undulations

$$N(P) = \frac{1}{\gamma_P} \sum_{i=1}^{K} \frac{m_i^{(n)}}{r_P^{n+1}} v_n^i(P) \quad , \tag{2}$$

and for gravity anomalies

$$\Delta g(P) = \sum_{i=1}^{K} \frac{m_i^{(n)}}{r_P^{n+2}} g_n^i(P) \quad . \tag{3}$$

The base functions in series (3) are:

$$g_n^i(P) = s_i \frac{\partial v_n^i(P)}{\partial s_i} + (n-1) v_n^i(P) \quad . \tag{4}$$

where s_i is the relative geocentric distance of the multipole.

Please cite this paper as: Marchenko A.N., O. Abrikosov, P. O. Romanishin (1995) Improvement of the gravimetric geoid in the Ukraine area using absolute gravity data. In: Vermeer M. (ed.) Proceedings of the Session G4 “Latest Developments in the Computation of Regional Geoids”, XX General Assembly EGS, Reports of the Finnish Geodetic Institute, 95:7, pp. 19-22, MASALA, 1995, pp. 19-22

Base functions $v_n^i(P)$ and their derivatives with respect to s_i may be found from the following recurrence formulae (where ψ_{iP} is the geocentric spherical distance between points i and P):

$$\left.\begin{aligned} nq_i^2 v_n^i &= (2n-1)(\cos\psi_{iP} - s_i)v_{n-1}^i - (n-1)v_{n-2}^i \ , \\ v_0^i &= \frac{1}{q_i} \ , \end{aligned}\right\} \qquad (5)$$

$$q_i = \sqrt{1 + s_i^2 - 2s_i\cos\psi_{iP}} \ . \qquad (6)$$

$$\left.\begin{aligned} nq_i^2 \frac{\partial v_n^i}{\partial s_i} &= (2n-1)(\cos\psi_{iP} - s_i)\frac{\partial v_{n-1}^i}{\partial s_i} - \\ &\quad -(n-1)\frac{\partial v_{n-2}^i}{\partial s_i} + 2n(\cos\psi_{iP} - s_i)v_n^i - \\ &\quad -(2n-1)v_{n-1}^i \ , \\ \frac{\partial v_0^i}{\partial s_i} &= \frac{\cos\psi_{iP} - s_i}{q_i^3} \ , \end{aligned}\right\} \qquad (7)$$

As a result of gravity data approximation in the frames of SMA technique, we obtain a finite set of radial multipoles that represent initial data with an appropriate accuracy. The corresponding computational procedure (SMA algorithm) for gravity field sequential approximation was described in detail in (Marchenko and Abrikosov 1994b). As it follows from that study, the SMA technique allows to obtain the require approximation accuracy even without applying any total least-squares adjustment to a whole set of multipole moments $\{m_i^{(n)}\}$. It is naturally, that final total readjustment is necessary for best optimization of constructing gravity model.

Thus, the general approach of radial multipole potentials applications for approximation of the Earth's gravity field in some (local, regional or global) area includes the following steps:

1. Preliminary construction of a gravity model applying the SMA technique to initial gravity data;
2. Final total readjustment of multipole moments $\{m_i^{(n)}\}$ for the whole set of previously obtained radial multipoles.

Data

In the present study the following data sets were used for geoid construction:

1. 768 free-air gravity anomalies averaged by blocks 5.0'×7.5' in the West Ukraine area;
2. 980 free-air gravity anomalies averaged by blocks 20'×30' in the remaining part of Ukraine;
3. 977 Geosat altimeter data distributed in the Black and Azov Seas aquatories.

Table 1. Characteristics of the gravity anomalies data sets.

Data set	Number of data	Resolution	Statistics (mGal) min.	max.	mean
West Ukraine	768	5.0'×7.5'	-73.5	67.4	-6.62
Ukraine	980	20'×30'	-66.8	158.4	5.86
Black and Azov Seas	977	Geosat traces	-79.2	87.0	5.74

Note here that altimeter data sets are distributed accordingly to Geosat traces. On the one hand, spherical distances between data points within a trace are equal near 3'. On the other hand, the maximum spherical distances between traces are equal near 1°. Thus we will deal with possible difficulties caused by heterogeneity of using data sets (gravity anomalies and altimetry) and specific irregularity of altimeter data spacing.

In order to exclude heterogeneity of using data sets we decide to carry out preprocessing altimeter data. As result, from Geosat data set of 977 gravity anomalies (located at the same points) were found on the ground of Tikhonov's regularization technique. In addition we used construction of local empirical and analytical covariance functions (derived from radial multipoles potentials for geoid heights that were replaced here by the Sea Surface Heights), which provided the covariance propagation to the gravity anomalies (Marchenko and Abrikosov 1994c) by means of least-squares collocation technique.

Thus in the further geoid determination we used 3 sets of gravity anomalies. Table 1 illustrates main characteristics of them.

Results of altimetry-gravimetric geoid determination

For geoid determination the traditional "remove-restore" procedure was realized in the following way.

The contribution of a global gravity model with appropriate resolution was removed from the initial gravity anomalies. For this reason new Earth gravity model (NFME) was used that consists from the set of (***5** central* [n=0,2,4,6,8] + *1417 non-central radial* [n>0]) multipole potentials (Abrikosov and Marchenko 1995). Note here, that on the one hand this model was constructed specially as normal field model for the *mathematically homogeneous* both regional and local field representations by technique performed. On the other hand, NFME normal field leads to the beautiful approximation (with decreasing of time expances for computation ~10 times) of the OSU91A (180 x 180) version for whole European region especially.

Table 2. Residuals of gravity anomalies (in mGal) in the analysed areas for various gravity models.

Residuals of gravity anomalies for model	Minimum	Maximum	Mean	R.m.s.	Standard deviation
(Initial local data set) - NFME	-78.79	103.98	4.36	26.28	25.92
(Initial local data set) - UFM	-5.81	5.28	0.05	1.67	1.67
(Initial local data set) - UFMC	-5.94	5.45	0.10	2.12	2.12

Then the residual gravity anomalies for terrestrial and marine areas were approximated by the set of 296 radial multipoles. We used sequential (SMA) approximation only without any joint adjustment of multipole moments. Besides, *we used multipoles with degrees n>0 only* (i.e. starting from radial dipoles) due to the well-known approximation's requirement of the physical geodesy (Moritz, 1980). Thus, *all obtained sets of radial multipoles do not contain zero degree multipoles (point masses)*. The r.m.s. of fit value ±2 mGal was the criterion of approximation process completion because the initial terrestrial (5.0'×7.5') gravity data accuracy was near ±2 mGal ÷ ±3mGal.

On the first step geoidal undulations were restored on the basis of combined gravity model consists of the global model NFME normal field and the "local" set of 296 non-central radial multipoles. The *whole set UFM consists from 1713 radial multipoles and 5 central multipoles describes the GRS80 reference ellipsoid field* (see Table 2).

On the second step new combine solution for all above mentioned initial Δg data was found with using of ***7** stations of gravity network that located on the seashore. These stations were chosen with the aim of hard control of our local gravity model*. Readjustment of 296 multiple moments was made with the additional conditions that making the gravity generated by the UFM model equal exactly to the gravity measured in 7 mentioned gravimetric stations. As a result we have the *new model UFMC that provides common agreement of terrestrial gravimetric, altimeter and gravity network data in the terrestrial/marine area*. Geoid solution UFMC for the Ukraine area and the gravity network stations that located on the seashore are shown in the Figure 1 (see Table 2 also).

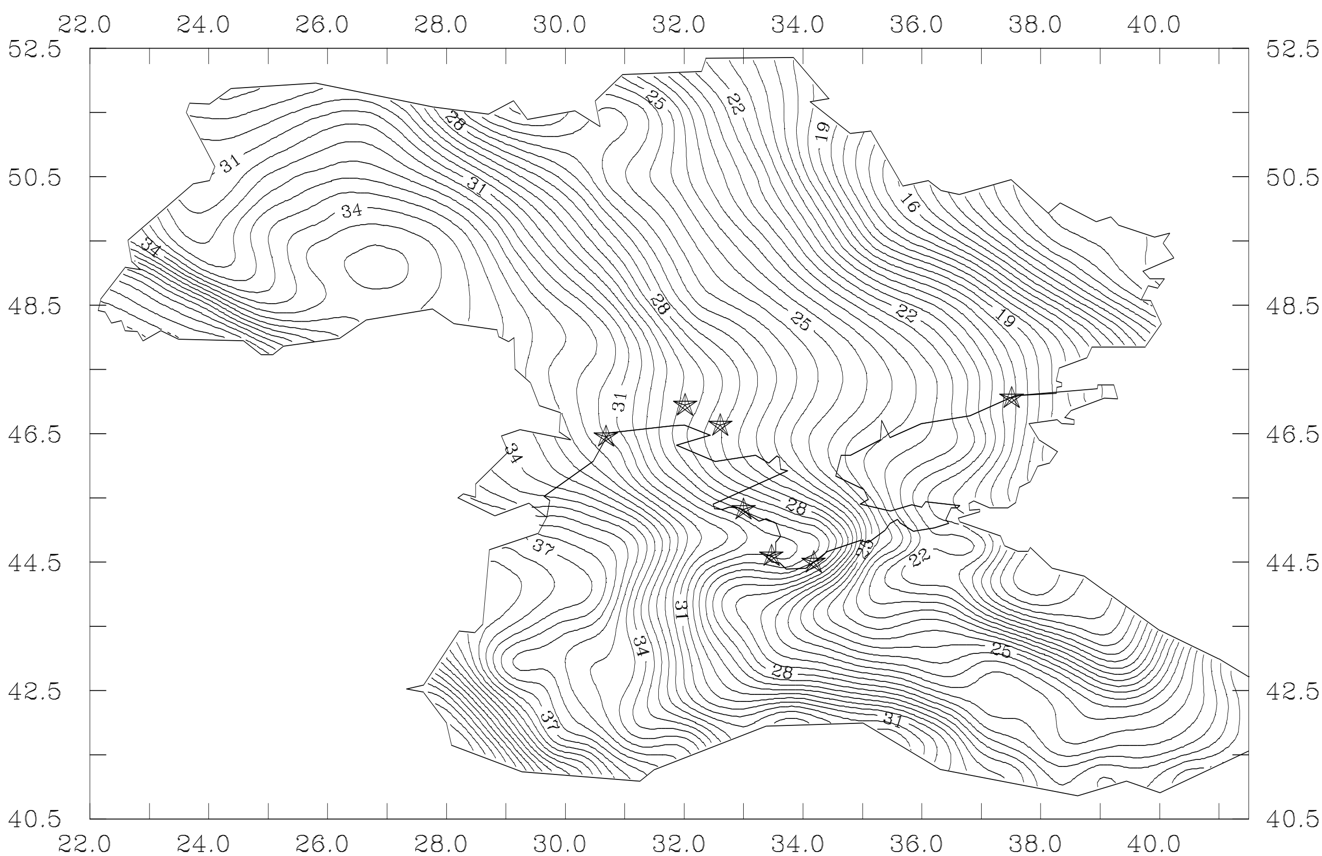

Fig. 1. Geoid solution UFMC for the Ukraine area referred to the GRS80 ellipsoid (contour interval: 0.5 m; gravimetric stations are shown by asterisks).

Table 3. Differencies between gravity models UFM and UFMC.

Characteristics of residuals	Geoidal undulations (m)	Gravity anomalies (mGal)
Minimum	-0.18	-0.57
Maximum	0.14	0.49
Mean	0.00	0.05
R.m.s.	0.02	0.45
Standard deviation	0.02	0.43

Conclusions

Thus, the sequential multipole analysis technique was used again for altimetry - gravimetric geoid construction in regional and local scales.

As a result, on the preliminary step *from Geosat data set the gravity anomalies (located at the same points) were found on the ground of least-squares collocation/ Tikhonov's regularization techniques*.

Two *altimetry - gravimetric geoid* solutions (models) were obtained:

1. The first model - *UFM* - approximates gravity anomalies in the studying terrestrial/marine area with accuracy ±2 mGal.
2. The second model - *UFMC* - approximates gravity anomalies in the studying terrestrial/marine area with the same accuracy ±2 mGal (see Table 2).

Differencies between gravity models *UFM* and *UFMC* are shown in Table 3.

Comparision of our *UFMC* reginal model with Geosat SSH values leads to the standard deviation of differencies equal 1.2 m and comparision (at the same points) of *OSU91A* (360x360) model with Geosat SSH values leads to the standard deviation of differencies equal 1.8 m. Finally we note that *UFMC* solution was computed by simultaneous adjustment of the preliminary geoid version *UFM* and the data of the fundamental gravity network of Ukraine taking seashore points of this network as the control ones.

Acknowlegments. Our deepest thanks to the International Geoid Service and the Ohio State University for help in receiving of Geosat data, especially to Prof. F.Sanso, and Prof. R.Rapp.

References

Abrikosov, O.A. and Marchenko, A.N. (1995). Construction of the Normal Field for the Mathematically Homogeneous Modeling of Regional Geoids by Radial Multipoles. *Presented at the XX General Assebbly EGS, Session G4: "Latest Development in the Computation of Regional Geoids"*, Hamburg, Germany, 1995.

Marchenko, A.N. and Abrikosov, O.A. (1994a). Preliminary geoid solution for Ukraine region. *Paper presented at the International Symposium "Geodynamics of Europe Mountain Systems"*, Lviv-Yaremcha, Ukraine, 1994.

Marchenko, A.N. and Abrikosov, O.A. (1994b). Geoid in the West Ukraine Area Derived by Means of Non-Central Multipole Analysis Technique. *Presented at the International Symposium "Gravity and Geoid"*, Graz, Austria, 1994.

Marchenko, A.N. and Abrikosov, O.A. (1994c). Covariance Functions Set Derived from Radial Multipole Potentials: Theory and Some Results for Regional Gravity Field in Central Europe. *Paper presented at the International Symposium "Gravity and Geoid"*, Graz, Austria, 1994.

Moritz, H. (1980). *Advanced Physical Geodesy*, S.L.Wichmann.